Let X, Z, Y be three variables with support $\{0,1\}$. Let f be a three-dimensional vector valued function $f : \{0,1\} \times \{0,1\} \times \{0,1\} \to \{0,1\} \times \{0,1\} \times \{0,1\}$

$$f := \begin{cases} 1\{a - b(Y + Z) \geq 0\} \\ 1\{c - b(X + Z) \geq 0\} \\ 1\{d - b(X + Y) \geq 0\} \end{cases}$$

Let $a, c, d \in R$ and $b > 0$. I have shown that the function f has at least one fixed point for any $a, c, d \in R$ and $b > 0$ and that the set of fixed points is finite. This implies that the set of fixed points has a maximum and a minimum. Now, I need to write an algorithm that finds the maximum and minimum fixed points for any value of $a, c, d \in R$ and $b > 0$. The problem is that the function is monotone decreasing in $\{0,1\} \times \{0,1\} \times \{0,1\}$ since $b > 0$. Could you help me?

To compare vectors, I can use coordinatewise comparison

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} \leftrightarrow X \geq X' \text{ and } Y \geq Y' \text{ and } Z \geq Z'$$

or lexicographical order

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \geq \begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} \leftrightarrow X > X' \text{ or } X = X' \text{ and } Y > Y' \text{ or } X = X' \text{ and } Y = Y' \text{ and } Z \geq Z'$$

Thank you very much!